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Discussing string extensions of the Standard Model in D brane world

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Abstract

In this talk we will describe the problems that one encounters when one tries to connect string theory with particle phenomenology. Then, in order to have chiral matter describing quarks and leptons, we introduce the magnetized D branes. Finally, as an explicit example of a string extension of the Standard Model, we will describe the one constructed by Ibáñez, Marchesano and Rabadán.

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Contents

1 String theory and experiments

The strongest motivation for string theory is the fact that it provides a consistent quantum theory of gravity unified with gauge interactions. This is a consequence of the fact that string theory has a parameter α', related to the string tension by $T = \frac{1}{2\pi\alpha'}$, of dimension of a $(length)^2$ that acts as an ultraviolet cutoff $\Lambda = \frac{1}{\sqrt{\alpha'}}$. Because of this physical cutoff all loop integrals are finite in the UV.

The presence of this dimensional parameter α' implies that string theory can be viewed as an extension, rather than an alternative, to field theory pretty much as special relativity and quantum mechanics are an extension of respectively, Galilean and Classical mechanics. Those latter theories can be obtained from the former ones by taking respectively the limit $c \to \infty$ and $h \to 0$. Analogously , by taking the zero slope limit ($\alpha' \to 0$), one can see that perturbative string theory reduces to a perturbative gauge field theory unified with an extension of Einstein's theory of general relativity and that, in this limit, one recovers the UV divergences of perturbative quantum gravity unified with gauge theories. As it is known since long time, they are due to the point-like structure of the elementary constituents [1].

But, if string theory has something to do with Nature, how can we see stringy effects in experiments? The answer to this question depends of course on the energy E available. If $\alpha' E^2 << 1$, then one will see only the limiting field theory. In other words, if we knew the strength of α' we could tell at what energy one would see stringy effects that will manifest as deviations from the field theoretical behaviour based on point-like objects. If $\frac{1}{\sqrt{\alpha'}} \sim 10\,TeV$, then stringy effects can be seen in present experiments, while, if $\frac{1}{\sqrt{\alpha'}} >> 10 TeV$, then the presence of a string theory cannot be directly seen.

Having said this, let us now discuss where we stand in string theory. The simplest string theory is the bosonic string that is, however, not immediately consistent because it contains tachyons in the spectrum. Around 1985 it was clear that we have 5 ten-dimensional consistent string theories: IIA, IIB, I, Heterotic $E_8 \times E_8$ and Heterotic $SO(32)$. They are inequivalent in string perturbation theory ($g_s < 1$), supersymmetric and unify in a consistent quantum theory gauge theories with gravity. It must be said, however, that, unlike α', the string coupling constant g_s is not a parameter to be fixed from experiments. In fact, it corresponds to the vacuum expectation value of a string excitation, called the dilaton, $g_s = \mathrm{e}^{\langle\phi\rangle}$, that should

be fixed by the minima of the dilaton potential. But the potential for the dilaton is flat in any order of string perturbation theory and therefore, for each value of $\langle\phi\rangle$, we have an inequivalent theory. This is clearly unsatisfactory for a theory, as string theory, that has the potential to explain everything. This is, however, not the only problem! In fact, if string theory is the fundamental theory unifying all interactions, why do we have 5 theories instead of just one? The key to solve this problem came from the discovery in string theory of new p-dimensional states, called D(irichlet) p branes [2]. In the following we will explain their origin.

The spectrum of massless states of the II theories is given in the table

$G_{\mu\nu}$	$B_{\mu\nu}$	ϕ	NS-NS sector	
Metric	Kalb-Ramond	Dilaton		
C_0, C_2	C_4, C_6	C_8	RR sector	IIB
C_1, C_3	C_5	C_7	RR sector	IIA

where the RR C_i stands for an antisymmetric tensor potential $C_{\mu_1\mu_2\ldots\mu_i}$ with i indices.

These antisymmetric potentials are generalizations of the electromagnetic potential A_μ

$$\int A_\mu dx^\mu \Longrightarrow \int A_{\mu_1\mu_2\ldots\mu_{p+1}} \frac{dx^{\mu_1}\wedge dx^{\mu_2}\cdots\wedge dx^{\mu_{p+1}}}{(p+1)!} \tag{1}$$

In fact, as the electromagnetic field is coupled to point-like particles, so they are coupled to p-dimensional objects.

There exist classical solutions of the low-energy string effective action that are coupled to the metric, the dilaton and are charged with respect a RR field. The RR potential behaves as

$$C_{01\ldots p} \sim \tfrac{1}{r^{d-3-p}} \Longleftrightarrow C_0 \sim \tfrac{1}{r} \quad \text{if } d=4, p=0$$

that generalizes to a p-dimensional object in a d-dimensional space-time the behaviour of the Coulomb potential valid for a point-particle in four dimensions. They are additional non-perturbative states of string theory with tension and RR charge given by:

$$\begin{aligned} \tau_p &= \frac{Mass}{p-volume} = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} \\ \mu_p &= \sqrt{2\pi}(2\pi\sqrt{\alpha'})^{3-p} \end{aligned} \tag{2}$$

Being non-perturbative objects their tension diverges in the perturbative limit ($g_s \to 0$).

They are called D(irichlet) p branes because they have open strings attached to their (p+1)-dim. world-volume:

$$\begin{aligned} \partial_\sigma X^\mu(\sigma=0,\pi;\tau) &= 0 \quad \mu=0\ldots p \\ \partial_\tau X^i(\sigma=0,\pi;\tau) &= 0 \quad i=p+1\ldots 10 \end{aligned} \tag{3}$$

As follows from the previous equations, the open string satisfies Neumann boundary conditions along the world-volume of the Dp brane and Dirichlet boundary conditions in the directions transverse to the world-volume of the Dp brane. Remember that the motion of a string is described by the string coordinate $X^{\mu}(\sigma,\tau)$ that is a function of the parameters σ and τ that parameterize the world-sheet of a string. This parameterization is such that $\sigma = 0, \pi$ correspond to the two end-points of an open string. It turns out that, in a brane world, the states corresponding to the excitations of open strings live in the (p+1)-dim. world-volume of a Dp brane, while those corresponding to the excitations of closed strings live in the entire ten dimensional space. This means that the gauge theories, described by open strings, live on the world-volume of a Dp brane, while gravity, described by closed strings, lives in the entire ten-dimensional space-time. In particular, if we have a stack of N parallel D branes, then we have N^2 open strings having their endpoints on the D branes: these are the degrees of freedom of the adjoint representation of $U(N)$. One concludes that the open strings attached to the same stack of D branes transform according to the adjoint representation of $U(N)$. The massless string excitations correspond to the gauge fields of $U(N)$. Therefore a stack of N D branes has a $U(N) = SU(N) \times U(1)$ gauge theory living on their world-volume.

The discovery of Dp branes opened the way in 1995 to the discovery of the string dualities and this led to understand that the 5 string theories were actually part of a unique 11-dimensional theory, called M theory.

However, in the experiments we observe only 4 and not 10 or 11 non-compact directions. Therefore 6 of the 10 dimensions must be compactified and small: $R^{1,9} \rightarrow R^{1,3} \times M_6$, where M_6 is a compact manifold. In order to preserve at least $N = 1$ supersymmetry M_6 must be a Calabi-Yau manifold. But this means that the low-energy physics will depend not only on α' and g_s , but also on the size and the shape of the manifold M_6.

Originally the most promising string theory for phenomenology was considered the Heterotic $E_8 \times E_8$ that was studied intensively. But in this theory both the fundamental string length $\sqrt{\alpha'}$ and the size of the extra dimensions are of the order of the Planck length:

$$\begin{aligned} &\frac{1}{\sqrt{\alpha'}} \equiv M_s = \frac{M_{Pl.}\sqrt{\alpha_{GUT}}}{2} \sim \frac{M_{Pl.}}{10} \\ &\frac{R}{\sqrt{\alpha'}} \sim 1 \;\; ; \;\; if \;\, g_s < 1 \end{aligned} \tag{4}$$

They are both too small to be directly observed in present and even future experiments! This means that, if we want to compare perturbative heterotic string theory with experiments, we need to have a very good control of the theory to be able to extrapolate to low energy.

Later on in 1998 it became clear that in type I and in a brane world one could allow for much larger values for the string length $\sqrt{\alpha'}$ and for the size of the extra

dimensions without being in contradiction with the experimental data [3]. However, it is not clear if Nature likes these larger values.

When we compactify 6 of the 10 dimensions, in addition to the dilaton, we generate a bunch of scalar fields (moduli) corresponding to the components of the metric and of the other closed string fields in the extra dimensions. Their vacuum expectation values, corresponding to the parameters of the compact manifold, are not fixed in any order of perturbation theory because their potential is flat. We get a continuum of string vacua for any value of the moduli and this is obviously not good for phenomenology. In order to compare string theory with particle phenomenology one needs to find mechanisms to stabilize the moduli. In the last few years a lot of progress has been made in this direction because one has been able to stabilize them by the introduction of non-zero fluxes for some of the NS-NS and R-R fields.

But we are still left with a discrete (and huge) quantity of string vacua and this problem goes under the name of the "Landscape Problem".

The question is now: how do we fix the vacuum we live in? Do we need to rely on the Anthropic principle or maybe can this problem be solved by reaching a better understanding of string theory?

Rather than to discuss these two alternatives it is, in my opinion, more useful to try to construct string extensions of the Standard model (SM) and of the Minimal Supersymmetric Standard Model (MSSM). This has been called a bottom-up approach because one does not derive the SM or the MSSM from string theory as one would do in a more ambitious top-down approach, but instead, since we know that the SM correctly describes Nature at the energy reached up to now, one tries to see if the SM can be consistently incorporated in string theory.

If we want to construct string extensions of the SM in an explicit way we must limit ourselves to toroidal compactifications with orbifolds and orientifolds and, most important, we need to have massless open strings corresponding to chiral fermions in four dimensions for describing quarks and leptons.

The simplest explicitly solvable models with chiral matter in four dimensions are those based on several stacks of intersecting branes or of their T-dual magnetized branes on $R^{3,1} \times T^2 \times T^2 \times T^2$. These are the models that we are going to describe in the following section.

2 Magnetized D branes

Magnetized branes are characterized by having a non-zero constant magnetic field along the six compact directions of the torus $T^2 \times T^2 \times T^2$. Let us assume that we have two stacks of magnetized D branes that we call stack a and stack b. We want to study the motion and the spectrum of the open strings attached with one end-point to the stack a and with the other end-point to stack b when the magnetizations on the two stacks are different from each other. This kind of open strings are called twisted, dycharged or chiral strings. Their motion is described by the following

action:

$$S = S_{bulk} + S_{boundary} \tag{5}$$

where

$$S_{bulk} = -\frac{1}{4\pi\alpha'}\int d\tau \int_0^\pi d\sigma \left[G_{ab}\partial_\alpha X^a \partial_\beta X^b \eta^{\alpha\beta} - B_{ab}\epsilon^{\alpha\beta}\partial_\alpha X^a \partial_\beta X^b\right] \tag{6}$$

and

$$\begin{aligned} S_{boundary} &= -q_a \int d\tau A_i^{(a)}\partial_\tau X^i|_{\sigma=0} + q_b \int d\tau A_i^{(b)}\partial_\tau X^i|_{\sigma=\pi} = \\ &= \frac{q_a}{2}\int d\tau F_{ij}^{(a)} X^j \dot{X}^i|_{\sigma=0} - \frac{q_b}{2}\int d\tau F_{ij}^{(b)} X^j \dot{X}^i|_{\sigma=\pi} \end{aligned} \tag{7}$$

The two gauge field strengths $F_{ij}^{(a,b)}$ are constant and we choose the gauge in which the vector potentials are given by:

$$A_i^{(a,b)} = -\frac{1}{2}F_{ij}^{(a,b)} x^j \ . \tag{8}$$

The data of the torus T^2, called moduli, are included in the constant G_{ij}, that is the metric of the torus T^2, and B_{ij}, that is a background two-index antisymmetric Kalb-Ramond field. They are related to the complex and Kähler structures of the torus T^2:

$$\begin{aligned} U &\equiv U_1 + iU_2 = \frac{G_{12}}{G_{11}} + i\frac{\sqrt{G}}{G_{11}} \\ T &\equiv T_1 + iT_2 = B_{12} + i\sqrt{G} \end{aligned} \tag{9}$$

given by

$$\begin{aligned} G_{ij} &= \frac{T_2}{U_2}\begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix} \\ B_{ij} &= \begin{pmatrix} 0 & -T_1 \\ T_1 & 0 \end{pmatrix} \end{aligned} \tag{10}$$

They are the closed string moduli [1].

F is constrained by the fact that its flux is an integer:

$$\int Tr\left(\frac{qF}{2\pi}\right) = m \Longrightarrow 2\pi\alpha' q F_{12} = \frac{m}{n} \tag{11}$$

They are the open string moduli. The D brane is wrapped n times on the torus and the flux of F, on a compact space as T^2, must be an integer m corresponding to a magnetic charge.

[1] In this talk, for the sake of simplicity, we take $T_1 = 0$

The most general motion of an open string in this constant background can be explicitly determined and the theory can be explicitly quantized [4].

One gets a string extension of the motion of an electron in a constant magnetic field on a torus. Also in the string case the ground state is degenerate and the degeneracy is given by the number of Landau levels. One can also see that, when $\alpha' \to 0$, one goes back to the problem of an electron in a constant magnetic field.

The mass spectrum of the string states can be exactly determined and it is given by:

$$\alpha' M^2 = N_4^X + N_4^\psi + N_{comp.}^X + N_{comp}^\psi + \frac{x}{2}\sum_{i=1}^{3} \nu_i - \frac{x}{2} \tag{12}$$

where $x = 0$ for fermions (R sector) and $x = 1$ for bosons (NS sector) and

$$N_4^X = \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n \quad ; \quad N_4^\psi = \sum_{n=\frac{x}{2}}^{\infty} n \psi_n^\dagger \cdot \psi_n \tag{13}$$

$$N_{comp}^X = \sum_{r=1}^{3} \left[\sum_{n=0}^{\infty} (n+\nu_r) a_{n+\nu_r}^{\dagger(r)} a_{n+\nu_r}^{(r)} + \sum_{n=1}^{\infty} (n-\nu_r) \bar{a}_{n-\nu_r}^{\dagger(r)} \bar{a}_{n-\nu_r}^{(r)} \right] \tag{14}$$

$$N_{comp}^\psi = \sum_{r=1}^{3} \left[\sum_{n=\frac{x}{2}}^{\infty} (n+\nu_r) \psi_{n+\nu_r}^{\dagger(r)} \psi_{n+\nu_r}^{(r)} + \sum_{n=1-\frac{x}{2}}^{\infty} (n-\nu_r) \bar{\psi}_{n-\nu_r}^{\dagger(r)} \bar{\psi}_{n-\nu_r}^{(r)} \right] \tag{15}$$

where

$$\nu_r = \nu_r^{(a)} - \nu_r^{(b)} \quad ; \quad \tan \pi \nu_r^{a,b} = \frac{m_r^{(a,b)}}{n_r^{(a,b)} T_2^{(r)}} \tag{16}$$

$T_2^{(r)}$ is the volume of the r-th torus.

In the fermionic sector the lowest state is the vacuum state. It is a 4-dimensional massless chiral spinor. This is due to the fact that the ten-dimensional GSO projection reduces to the four-dimensional one because the fermionic zero modes are absent in the six-dimensional compact directions.

For generic values of ν_1, ν_2, ν_3 there is no massless state in the bosonic sector and this means that, in general, the original 10-dim supersymmetry is broken [5].

The lowest bosonic states are

$$\bar{\psi}_{\frac{1}{2}-\nu_r}^{\dagger(r)} |0> \quad ; \quad r = 1, 2, 3 \tag{17}$$

with masses, respectively given by

$$\alpha' M^2 = \frac{1}{2} \sum_{s=1}^{3} \nu_s - \nu_r \quad ; \quad r = 1, 2, 3 \tag{18}$$

and

$$\bar{\psi}^{\dagger(1)}_{\frac{1}{2}-\nu_1}\bar{\psi}^{\dagger(2)}_{\frac{1}{2}-\nu_2}\bar{\psi}^{\dagger(3)}_{\frac{1}{2}-\nu_3}|0> \tag{19}$$

with mass given by

$$\alpha' M^2 = \frac{2-\nu_1-\nu_2-\nu_3}{2} \tag{20}$$

One of these states becomes massless if one of the following identities is satisfied:

$$\begin{aligned} &\nu_1 = \nu_2+\nu_3 \ \ ; \ \ \nu_2 = \nu_1+\nu_3 \\ &\nu_3 = \nu_1+\nu_2 \ \ ; \ \ \nu_1+\nu_2+\nu_3 = 2 \end{aligned} \tag{21}$$

In each of these cases a four-dimensional $\mathcal{N}=1$ supersymmetry is restaured!

In general the ground state for the open strings, having their end-points, respectively on stacks a and b, is degenerate.

Its degeneracy is given by the number of Landau levels as in the case of a point-like particle:

$$I_{ab} = -\prod_{i=1}^{3}\left\{ n_i^{(a)} n_i^{(b)} \int \left[\frac{q_a F_i^{(a)} - q_b F_i^{(b)}}{2\pi} \right] \right\} = \prod_{i=1}^{3}\left[m_i^{(b)} n_i^{(a)} - m_i^{(a)} n_i^{(b)} \right] \tag{22}$$

that gives the number of families in the phenomenological applications. Note that I_{ab} can be both positive and negative. The convention is that a positive I_{ab} describes left-handed fermions, while a negative I_{ab} describes right-handed fermions.

In this section we have considered D9 branes on $T^2 \times T^2 \times T^2$ with three magnetizations ν_1, ν_2, ν_3. This system is T-dual to a system of non-magnetized, but intersecting D6 branes wrapped on the three one-cycles $[a_r]$ of the three tori with wrapping numbers n_r. In this T-dual picture the magnetizations become the angles in the three tori between the D6 branes and the number of Landau levels become the number of intersections. In the following, we will mostly use the language of the D6 branes instead of that of the magnetized branes.

3 An example of string extension of the Standard Model

In this section we briefly describe a consistent string extension of the SM constructed in Ref. [6]. In order to have a string extension of the SM we need to introduce four different stacks of magnetized D9 branes that we call: a, b, c, d. The stack a consists of three branes, called the baryonic branes, that have on their world-volume a $U(3) = SU(3) \times U(1)$ gauge theory, where the $SU(3)$ subgroup is the color $SU(3)$ describing strong interactions. The stack b consists of two branes, called the left

branes, that have on their world-volume a $U(2) = SU(2) \times U(1)$ gauge theory with the gauge group $SU(2)$ being the $SU(2)_L$ of the SM. The stacks c and d consist of one brane each, called respectively, the leptonic and the right branes. They have both a $U(1)$ theory living on their world-volume. In conclusion, on this system of branes we have the following gauge group:

$$SU(3)_a \times SU(2)_b \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d \tag{23}$$

where $SU(3)_a$ is the color $SU(3)$ and $SU(2)_b$ is the $SU(2)_L$. In addition to these two groups of the SM we have four $U(1)$ instead of the just the $U(1)$ hypercharge. In order to determine the $U(1)$ hypercharge and to see the role of the other $U(1)$'s we need first to discuss the cancellation of the gauge anomalies. In fact, in a model, like the one discussed above, containing chiral fermions, we need to check that all the gauge anomalies are cancelled. This cannot be realized in the model just discussed above, but we have to add to it an orientifold projection. This means that for each D brane we need to introduce its image that we denote with a star. Therefore for each of the four stacks of D branes a, b, c, d we will have their images denoted with a^*, b^*, c^*, d^*.

In our orientifold theory with intersecting D6 branes the tadpole cancellation condition reads [2]:

$$\sum_{A=1}^{K} N_A \left(\Pi_A + \Pi_{A^*}\right) = 32\Pi_{O6} \tag{24}$$

In the case of the torus $T^2 \times T^2 \times T^2$ the three-cycles $\Pi_A, \Pi_{A^*}, \Pi_{O6}$ are given by:

$$\begin{aligned} \Pi_A &= \prod_{r=1}^{3} \left[n_A^{(r)}[a_r] + m_a^{(r)}[b_r] \right] \\ \Pi_{A^*} &= \prod_{r=1}^{3} \left[n_A^{(r)}[a_r] - m_A^{(r)}[b_r] \right] \\ \Pi_{O6} &= \prod_{r=1}^{3} [a_r] \end{aligned} \tag{25}$$

where $[a_r]$ and $[b_r]$ are the two cycles of the r-th torus satisfying the following relations:

$$\begin{aligned} [a_r] \cdot [a_s] &= [b_r] \cdot [b_s] = 0 \\ [a_r] \cdot [b_s] &= -[b_r] \cdot [a_s] = \delta_{rs} \end{aligned} \tag{26}$$

[2]Remember that in this talk we take $T_1 = 0$.

Using the previous equations it is possible to compute the following quantities:

$$\Pi_A \cdot \Pi_B = I_{AB} \equiv \prod_{r=1}^{3} \left[n_A^{(r)} m_B^{(r)} - n_B^{(r)} m_A^{(r)} \right]$$
$$\Pi_A \cdot \Pi_{O6} = - \prod_{r=1}^{3} m_A^{(r)} = I_{AO6} \tag{27}$$

and

$$\Pi_A \cdot \Pi_{B^*} = -\Pi_{A^*} \cdot \Pi_B \equiv I_{AB^*} = - \prod_{r=1}^{3} \left[n_A^{(r)} m_B^{(r)} + n_B^{(r)} m_A^{(r)} \right] \tag{28}$$

Before we proceed, let us summarize the spectrum of open strings stretched between intersecting D6 branes [7]. The open strings attached with one end-point at the stack A and with the other end-point at the stack B transform according to the bifundamental representation $(N_A, \bar{N}_B)$ of the gauge groups $U(N_A)$ and $U(N_B)$ respectively, and their number is equal to $I_{AB} = \Pi_A \cdot \Pi_B$:

$$(A, B) \rightarrow (N_A, \bar{N}_B)$$
$$I_{AB} = \prod_{r=1}^{3} \left[n_A^{(r)} m_B^{(r)} - n_B^{(r)} m_A^{(r)} \right] \tag{29}$$

The massless chiral fermions corresponding to open strings stretched between a stack A and the image B^* of the stack b transform according to the bifundamental representation (N_A, N_B) of the gauge groups $U(N_A)$ and $U(N_B)$ and their number is equal to I_{AB^*}:

$$(A, B^*) \rightarrow (N_A, N_B)$$
$$I_{AB^*} = - \prod_{r=1}^{3} \left[n_A^{(r)} m_B^{(r)} + n_B^{(r)} m_A^{(r)} \right] \tag{30}$$

Finally the massless chiral fermions corresponding to open strings having one end-point attached to the stack A and and the other one to its image A^* transform according to both the two-index symmetric and two-index antisymmetric representation of the gauge group $U(N_A)$. Their total number is given by

$$I_{AA^*} = -8 \prod_{r=1}^{3} n_A^{(r)} m_A^{(r)} \tag{31}$$

Their multiplicity is given respectively by:

$$A = -4 \prod_{r=1}^{3} m_A^{(r)} \left(\prod_{r=1}^{3} n_A^{(r)} + 1 \right)$$
$$S = -4 \prod_{r=1}^{3} m_A^{(r)} \left(\prod_{r=1}^{3} n_A^{(r)} - 1 \right) \tag{32}$$

They can also written as:

$$
\begin{aligned}
A &= \frac{1}{2}\left(I_{AA^*} + N_{O6} I_{AO6}\right) \\
S &= \frac{1}{2}\left(I_{AA^*} - N_{O6} I_{AO6}\right)
\end{aligned} \tag{33}
$$

Notice that the $(A,S)_A$ representations are only coupled to a single gauge group $U(N_A)$, while the bi-fundamentals are coupled to two gauge groups.

Having discussed the spectrum of chiral fermions we can now compute the coefficient of the non-abelian anomaly. We follow the notation of Ref. [8] where more details can be found. The non-abelian anomaly is given by:

$$
\begin{aligned}
&\mathcal{A}_{(SU(N_A);SU(N_B)} \equiv \sum_r A(r) = \sum_{B\neq A} N_b (I_{AB} + I_{AB^*}) + \\
&+\frac{1}{2}\left(I_{AA^*} + N_{O6} I_{AO6}\right)(N_A - 4) + \\
&+\frac{1}{2}\left(I_{AA^*} - N_{O6} I_{AO6}\right)(N_A + 4)
\end{aligned} \tag{34}
$$

where the three contributions come respectively, from the fermions in the fundamental, in the two-index antisymmetric and in the two-index symmetric representations of the gauge group $SU(N_A)$. $A(r)$ is related to the cubic Casimir and is given in Table (4.1) of Ref. [8]. The previous expression can be written as follows:

$$
\begin{aligned}
&\mathcal{A}_{(SU(N_A);SU(N_B)} = \\
&= \sum_B N_B (I_{AB} + I_{AB^*}) - 4 N_{O6} I_{AO6} = \Pi_A \times \\
&\times \left[\sum_B N_B \left(\Pi_B + \Pi_{B^*}\right) - 32 \Pi_{O6}\right] = 0
\end{aligned} \tag{35}
$$

where $N_{O6} = 8$. In the last step of the previous equation we have used Eq.s (27) and (28) and we have imposed the tadpole cancellation condition in Eq. (24). In conclusion, if we impose the tadpole cancellation condition we have automatically eliminated the non-abelian anomalies.

The coefficient of the mixed anomalies is given by:

$$
\begin{aligned}
&\mathcal{A}_{U(1)_A;SU(N_B)} \equiv \sum_r Q_A(r) C_B(r) = \frac{1}{2}\delta_{AB} \times \\
&\times \left(\sum_C (I_{AC} + I_{AC^*}) N_c - Q_{O6} N_{O6} I_{AO6}\right) + \\
&+\frac{1}{2} N_A (I_{AB} + I_{AB^*})
\end{aligned} \tag{36}
$$

where the $U(1)$ charge $Q_A(r)$ and the quadratic Casimir $C_B(r)$ are given again in Table (4.1) of Ref. [8]. If the tadpole cancellation condition is satisfied, then the first term in Eq. (36) is vanishing. We are left with the second term that we will discuss later on. This means that we have only fundamental representations with degeneracy equal to I_{AB}.

Finally the $U(1)$ anomalies are given by:

$$\begin{aligned}
&\mathcal{A}_{U(1)_A;U(1)_B^2} \equiv \sum_r Q_A(r) Q_B^2(r) = \frac{1}{3}\delta_{AB} N_A \times \\
&\times \left(\sum_C (I_{AC} + I_{AC^*}) N_C - Q_{O6} N_{O6} I_{AO6} \right) \\
&+\frac{1}{2} N_A N_B (I_{AB} + I_{AB^*})
\end{aligned} \tag{37}$$

The first term in the previous equations is vanishing if we impose the tadpole cancellation equation. We are left with the second term that we will analyze later.

Let us now go back to our model with four stacks of D6 brane introduced at the beginning of this section. Each of the stacks will be wrapping some cycles $[\Pi_A]$ $(A = a, b, c, d)$ of the product of three tori. They will intersect each other I_{AB} number of times. The chiral fermions live at each intersection and we can choose the various intersecting numbers in such a way to reproduce the spectrum of the Standard Model. We have seen that the open strings stretching between each brane and its image give rise to chiral fermions transforming according to the double symmetric and double antisymmetric representation. Those states do not appear in the Standard Model and therefore we have to impose that:

$$I_{aa^*} = I_{bb^*} = I_{cc^*} = I_{dd^*} = 0 \tag{38}$$

Since we have:

$$I_{AA^*} = -8 \prod_{r=1}^{3} \left[m_A^{(r)} n_A^{(r)} \right] = 0 \tag{39}$$

This condition can be satisfied if we choose:

$$\prod_{r=1}^{3} m_A^{(r)} = 0 \quad ; \quad A = a, b, c, d \tag{40}$$

It implies also that

$$I_{AO6} = 0 \tag{41}$$

as follows from Eq. (27). With the choice in Eq. (40) we have imposed that both A and S in Eq. (33) are zero. In this way we have imposed that there is

no chiral fermion transforming according to the double symmetric and the double antisymmetric representations, as it is the case in the SM.

We have to choose the other intersecting numbers in such a way to get the correct spectrum of the Standard Model with three families. This can be realized by imposing the following intersecting numbers:

$$\begin{aligned} I_{ab} &= 1 \quad &;\quad I_{ab^*} &= 2 \\ I_{ac} &= -3 \quad &;\quad I_{ac^*} &= -3 \\ I_{bd} &= -3 \quad &;\quad I_{bd^*} &= 0 \\ I_{cd} &= 3 \quad &;\quad I_{cd^*} &= -3 \end{aligned} \tag{42}$$

with all others being zero.

We now show that with this choice we cancel the non-abelian anomalies. They cancel if the following condition is satisfied:

$$\sum_B \left(I_{AB} + I_{AB^*}\right) N_B = 0 \quad ; \quad A = a, b \tag{43}$$

This follows from Eq. (35) together with Eq. (41). For $A = a$ we get:

$$\begin{aligned} &\sum_B \left(I_{aB} + I_{aB^*}\right) N_B = \\ &= \left(I_{ab} + I_{ab^*}\right) N_b + \left(I_{ac} + I_{ac^*}\right) N_c = \\ &= 2(1+2) - 3 - 3 = 0 \end{aligned} \tag{44}$$

while for $A = b$ we get (remember that $I_{ba} = -I_{ab}$ and $I_{ba^*} = I_{ab^*}$):

$$\sum_\beta \left(I_{b\beta} + I_{b\beta^*}\right) N_\beta = \left(I_{ba} + I_{ba^*}\right) N_a + I_{bd} N_d = 3(-1+2) - 3 = 0 \tag{45}$$

Note that, if we had chosed $I_{ab} = 3$ and $I_{ab^*} = 0$ we would not have satisfied the previous anomaly cancellation equation. This means that, from the point of view of the anomaly cancellation, the representations 2 and $\bar{2}$ are not equivalent. Furthermore, the anomaly cancellation requires that the number of generations be equal to the number of colors. In conclusion, with the choice in Eqs. (43) we do not have any non-abelian anomaly.

We have to check now what happens for the mixed $U(1)_A - SU(N_B)_B$ anomalies. The coefficient of these anomalies is equal to (see Eq. (36)):

$$\mathcal{A}_{AB} \equiv \frac{1}{2} N_A \left(I_{AB} + I_{AB^*}\right) \tag{46}$$

We have to compute this quantity for $B = a, b$. For $B = a$ we get:

$$\mathcal{A}_{ba} = 1 \quad ; \quad \mathcal{A}_{ca} = 0 \quad ; \quad \mathcal{A}_{da} = 0 \tag{47}$$

and for $B = b$ we get:

$$\mathcal{A}_{ab} = \frac{9}{2} \;\; ; \;\; \mathcal{A}_{cb} = 0 \;\; ; \;\; \mathcal{A}_{db} = \frac{3}{2} \tag{48}$$

They are the coefficients of the anomaly of the various $U(1)$ currents involving the second Chern class of both the non-abelian $SU(3)$ and $SU(2)$. For instance, the $U(1)$ current corresponding to Q_a has the $SU(2)$ anomaly, but not the $SU(3)$ anomaly, while that corresponding to Q_b has $SU(3)$ anomaly, but not $SU(2)$ anomaly. From the previous coefficients we can read that the $U(1)$'s with generators

$$Q_c \qquad and \qquad Q_a - 3Q_d \tag{49}$$

are anomaly free, while the orthogonal combinations:

$$\hat{Q} \equiv 3Q_a + Q_d \quad ; \quad Q_b \tag{50}$$

are anomalous. In terms of the two non-anomalous $U(1)$'s we can construct the generator of the $U(1)$ hypercharge:

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d \tag{51}$$

that must be anomaly-free. The three orthogonal combinations corresponding to

$$\begin{aligned} Q_b \quad ; \quad \hat{Q} = 3Q_a + Q_d \\ \tilde{Q} = Q_a + \frac{10}{3}Q_c - 3Q_d \end{aligned} \tag{52}$$

are instead anomalous. Remember that Q_a corresponds to the baryon number B, while Q_d corresponds to the lepton number L:

$$Q_a = 3B \quad ; \quad \hat{Q} = 3Q_a + Q_d = 3(B - L) \tag{53}$$

Q_b corresponds instead to a Peccei-Quinn symmetry that has a $SU(3)$ mixed anomaly as follows from the fact that $\mathcal{A}_{ba}$ in Eq. (47) is different from zero. However, at this point we have the problem that those $U(1)$ correspond to anomalous gauge symmetries . Furthermore, Q_a and $\hat{Q}$ are in our case gauge symmetries and not global symmetries as in the SM. In the following we will show how these two problems are solved.

We have seen that the $U(1)$ hypercharge in Eq. (51) is anomaly free, while the other three $U(1)$ are anomalous. In string theory the anomaly comes from the one-loop planar diagram that, in the field theory limit, reduces to the well known triangular diagram of anomalies. However, in string theory we have also a non-planar diagram that actually cancels the anomaly of the planar one [9]. In particular, this cancellation comes from a term of the non-planar diagram that corresponds to the exchange, in the closed string channel, of a RR C_2 field that is

coupled, on the one side, to the gauge field of anomalous $U(1)$ and on the other side to the two gauge fields of the non-abelian group. One of the couplings is divergent in the limit $\alpha' \to 0$, while the other goes to zero in such a way that their product is independent of α', giving a contribution that exactly cancels that of the planar diagram. This is called Green-Schwarz mechanism because it is the same that eliminates the gauge anomaly in ten-dimensional type I string theory. In conclusion, also the anomalies of the other three $U(1)$ cancel if one takes into account the contribution to the anomaly of the non-planar diagram. This is a pure stringy effect although it gives a contribution that is not vanishing in the field theory limit.

If this were the end of the story, then we will be left with three additional gauge $U(1)$'s and not with just one as in the SM.

However, this is not true because string theory contains an additional mechanism, discovered in string theory by Cremmer and Scherk [10], that is called Stückelberg mechanism, according to which the three extra $U(1)$ get a non-zero mass. This is again due to the coupling of the $U(1)$ gauge fields with a RR field C_2 that diverges as $\frac{1}{\sqrt{\alpha'}}$ that, together with the kinetic terms for the gauge field and for C_2, provides a non-zero mass to the $U(1)$ gauge field. In conclusion, the three extra $U(1)$ get a non-zero mass of the order $\frac{1}{\sqrt{\alpha'}}$ and this implies that the original three local $U(1)$ become three global ones. This is the origin of the global symmetries, $B-L$ and baryon number, of the SM in this string extension of the SM. Finally, we must be careful that the gauge boson of the hypercharge $U(1)$ does not get a mass. This can be done imposing an extra condition that can be found in Ref. [8] together with a more complete description of the model. In this way one obtains a string extension of the SM with only one additional particle, the right-handed neutrino.

The three $U(1)$'s whose gauge boson got a mass, are exact global symmetries at each order of string perturbation theory. Therefore the baryon and lepton numbers are exactly preserved and Majorana neutrino masses are not allowed at each order of perturbation theory. These symmetries, however, can be broken by instantons and this has been proposed as a way to give a Majorana mass to the neutrinos [11, 12]. It would be very interesting if this effect could be due to pure stringy effects that disappear in the field theory limit ($\alpha' \to 0$)!

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References

[1] L.D. Landau and E.M. Liftschitz, The classical theory of fields.

[2] J. Polchinski, Phys. Rev. Lett. 75:4724 (1995), hep-th/9510017.

[3] I. Antoniadis, N. Arkani-Hamed and S. Dimopoulos, Phys. Lett. B 436:257(1998), hep-ph/9804398.

[4] C. Bachas and M. Porrati, Phys.Lett. **B**296, 77 (1992), `hep-th/9209032`.

[5] C. Bachas, hep-th/9503030.

[6] L.E.Ibáñez, F. Marchesano and R. Rabadán, JHEP 0111:002 (2001), hep-th/0105155.

[7] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP 0010:006 (2000), hep-th/0007024.

[8] F. Marchesano, PhD thesis, hep-th/0307262.

[9] L.E.Ibáñez, R. Rabadán and A.M. Uranga, Nucl.Phys.B 542:112 (1999), hep-th/9808139.

[10] E. Cremmer and J. Scherk, Nucl. Phys. B 72, 117 (1974).

[11] R. Blumenhagen, M. Cvetic and T. Weigand, Nucl. Phys. B 771:113 (2007), hep-th/0609191.

[12] L.E.Ibáñez and A.M. Uranga, JHEP 0703:052 (2007), hep-th/0609213.